# Chapter 6 – Quadrilaterals

## 6.1 – Parallelograms on the Coordinate Plane

Objectives:
- Show that a quadrilateral is a parallelogram on the coordinate plane
- Identify and verify parallelograms

**DISTANCE FORMULA:**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**MIDPOINT FORMULA:**

$$(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

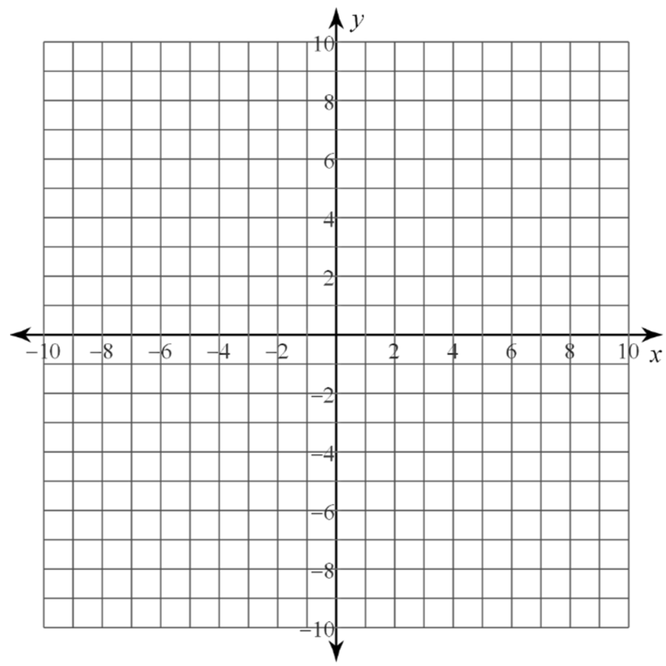
**SLOPE FORMULA:**

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- Investigation:
  - Plot your assigned parallelogram on the coordinate plane.
  - Find the following:
    - The slope of each side
    - The length of each side
    - The slope of both diagonals
    - The length of both diagonals
    - The midpoint of both diagonals

    *You may want to show your work on a separate sheet of paper.*

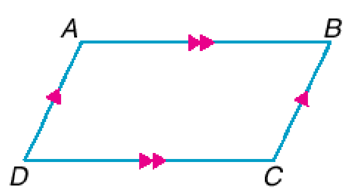
    *Record your findings in the table below.*




| SIDES: | | | |
|---|---|---|---|
| $m_{AB} =$ | $m_{BC} =$ | $m_{CD} =$ | $m_{AD} =$ |
| $AB =$ | $BC =$ | $CD =$ | $AD =$ |
| **DIAGONALS:** | | | |
| $m_{AC} =$ | $m_{BD} =$ | $AC =$ | $BD =$ |
| $AC(x_m, y_m) =$ | | $BD(x_m, y_m) =$ | |

- Observations:
  - What do you observe about the slopes of the sides of the parallelogram? What does this tell us?
  - What do you observe about the lengths of the sides of the parallelogram?
  - What do you observe about the slopes of the diagonals of the parallelogram?
  - What do you observe about the lengths of the diagonals of the parallelogram?
  - What do you observe about the midpoints of the diagonals of the parallelogram? What does this tell us?

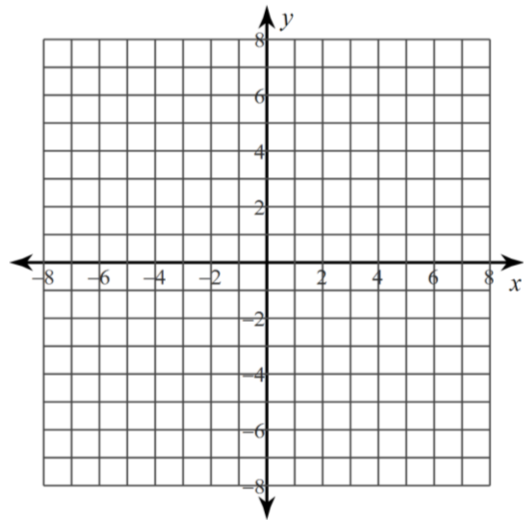
- Parallelograms
  - A <u>parallelogram</u> is a quadrilateral with both pairs of opposite sides parallel.
    - $\overline{AB} \parallel \overline{DC}$ & $\overline{AD} \parallel \overline{BC}$




- Proving Parallelograms on the Coordinate Plane
  - Show that both pairs of opposite sides are parallel
  - Show that both pairs of opposite sides are congruent
  - Show that ONE pair of opposite sides is both parallel AND congruent
  - Show that the diagonals bisect each other

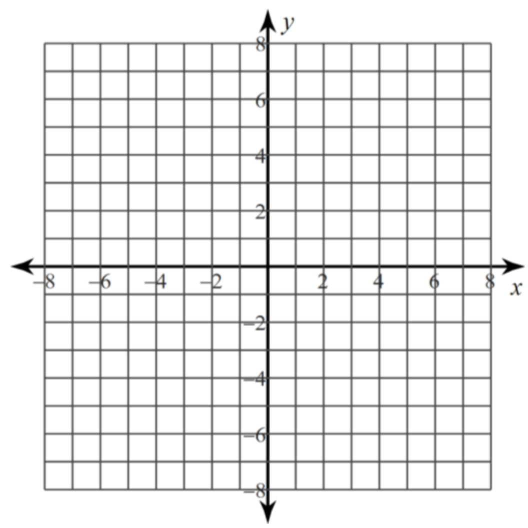
## EXAMPLES:

1. Three vertices of parallelogram *WXYZ* are $W(-1,-3), X(-3,2), \& Z(4,-4)$. Graph these vertices. Use slopes to find the coordinates of the vertex *Y*.



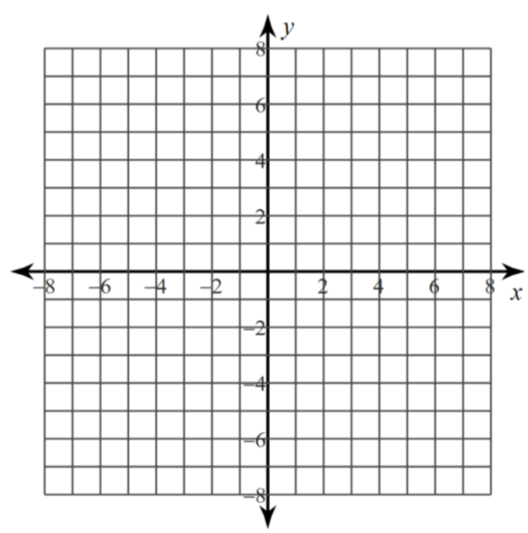

2. Show that quadrilateral *ABCD* is a parallelogram using the definition of parallelogram: *show that both pairs of opposite sides are parallel*.

   $A(-3,2), B(-2,7), C(2,4), \& D(1,-1)$



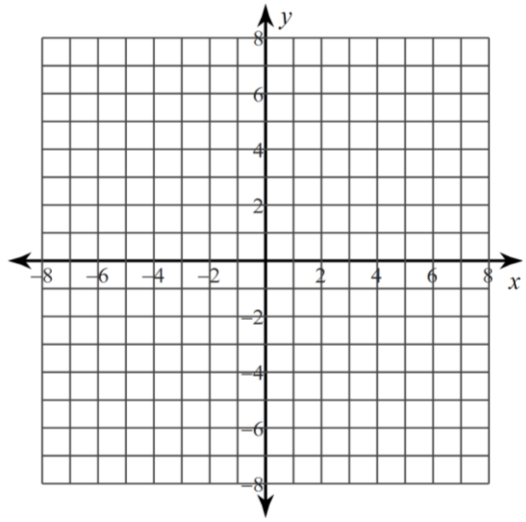

3. Show that quadrilateral *FGHJ* is a parallelogram by showing that *one pair of opposite sides are both parallel and congruent*.

   $F(-4,-2), G(-2,2), H(4,3), \& J(2,-1)$



4. Show that quadrilateral *KLMN* is a parallelogram by showing that *the diagonals bisect each other*.

$K(-3,0), L(-5,7), M(3,5), \& N(5,-2)$



# 6.2 – PROPERTIES OF PARALLELOGRAMS

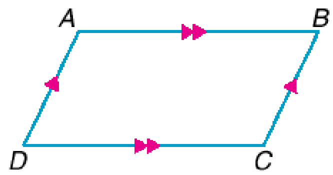
<u>Objectives:</u>

- Know and prove the properties of parallelograms
- Apply the properties of parallelograms to find side lengths, segment lengths, and angle measures

❖ Parallelograms

➢ A <u>parallelogram</u> is a quadrilateral with both pairs of opposite sides parallel.

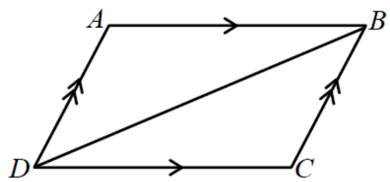
- $\overline{AB} \parallel \overline{DC}$ & $\overline{AD} \parallel \overline{BC}$




❖ Proving the Properties of Parallelograms

Given: *ABCD* is a parallelogram

Prove: $\angle A \cong \angle C$ & $\angle B \cong \angle D$

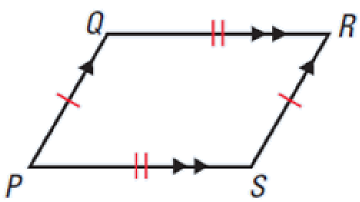



| STATEMENTS | REASONS |
| --- | --- |
| | |

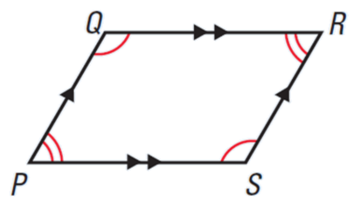
❖ Properties of Parallelograms

➢ If a quadrilateral is a parallelogram, then…

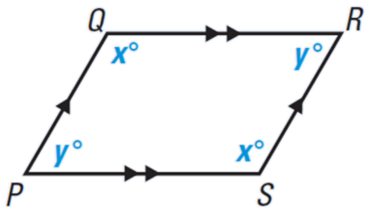
- Both pairs of opposite sides are congruent
  - $\overline{PQ} \cong \overline{SR}$ & $\overline{QR} \cong \overline{PS}$



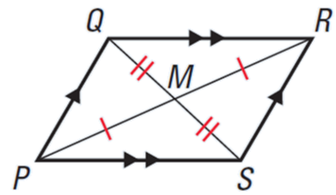

- Both pairs of opposite angles are congruent
  - $\angle Q \cong \angle S$ & $\angle P \cong \angle R$



- Consecutive angles are supplementary
  - $x° + y° = 180°$




- The diagonals bisect each other
  - $\overline{QM} \cong \overline{MS}$ & $\overline{PM} \cong \overline{MR}$




***The sum of all four angles in any quadrilateral is 360°.***

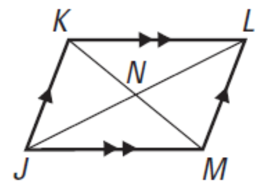
## EXAMPLES: USING THE PROPERTIES OF PARALLELOGRAMS

1. Complete each statement about *JKLM*.

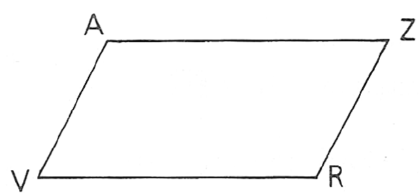
$\overline{JK} \cong$ ______ $\quad\quad \angle MLK \cong$ ______

$\angle JKL \cong$ ______ $\quad\quad \overline{JN} \cong$ ______

$\angle MNL \cong$ ______ $\quad\quad \overline{NM} \cong$ ______

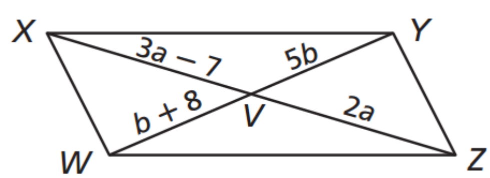



2. *VRZA* is a parallelogram.
Given: $AV = 2x - 4$
$VR = 3y + 5$
$RZ = \frac{1}{2}x + 8$
$ZA = y + 12$

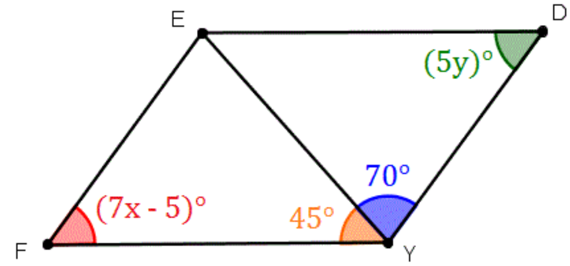



Find: The perimeter of *VRZA*

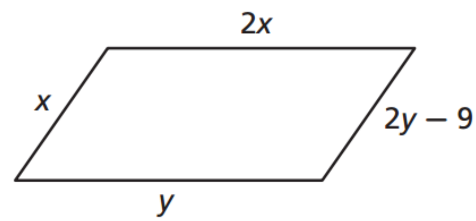
3. *WXYZ* is a parallelogram. Find each measure.
   a. *WV*
   b. *YW*
   c. *XZ*
   d. *ZV*



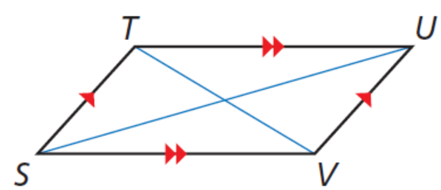
4. *FEDY* is a parallelogram. Find the value of each variable.




5. For the given parallelogram, set up and solve a system of equations to find the value of the variables.




6. In *STUV*, $m\angle TSU = 32°$, $m\angle USV = x^2$, $m\angle TUV = 12x$, and $\angle TUV$ is an acute angle. Find the value of $x$ (that makes sense) and $m\angle USV$.



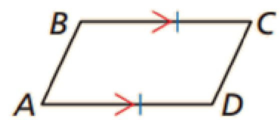
# 6.3 – Proving Quadrilaterals are Parallelograms

Objectives:

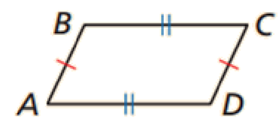
- Prove that a quadrilateral is a parallelogram
- Identify and verify parallelograms

❖ Conditions for Parallelograms

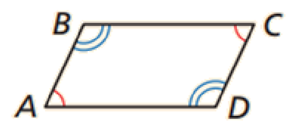
- If both pairs of opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram. (Definition)
- If one pair of opposite sides of a quadrilateral is parallel and congruent, then the quadrilateral is a parallelogram.
  - If $\overline{BC} \parallel \overline{AD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.



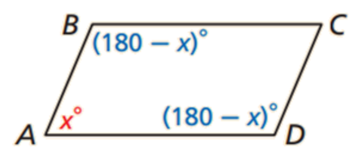

- If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
  - If $\overline{BC} \cong \overline{AD}$ and $\overline{AB} \cong \overline{CD}$, then $ABCD$ is a parallelogram.



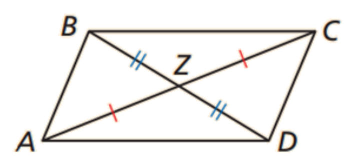

- If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
  - If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.




- If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram
  - If $\angle A$ is supplementary to $\angle B$ and $\angle A$ is supplementary to $\angle D$, then $ABCD$ is a parallelogram.



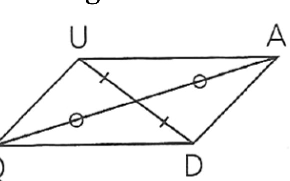

- If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
  - If $\overline{AZ} \cong \overline{ZC}$ and $\overline{BZ} \cong \overline{ZD}$, then $ABCD$ is a parallelogram.



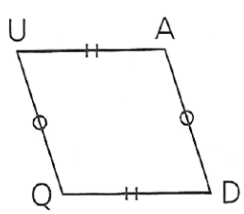

## Examples: Identifying Parallelograms

1. For each quadrilateral $QUAD$, state the property or definition that proves that $QUAD$ is a parallelogram.

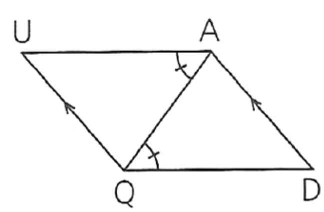
a



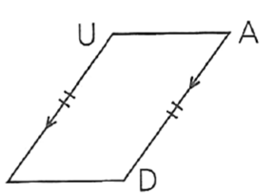

c



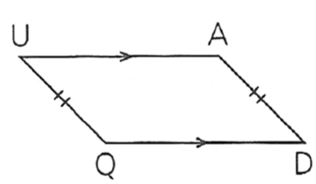

e




b




d

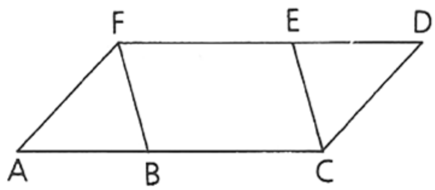


## EXAMPLES: PROVING PARALLELOGRAMS

2. Given: *ACDF* is a parallelogram

   $\angle AFB \cong \angle ECD$

   Prove: *FBCE* is a parallelogram




| STATEMENTS | REASONS |
|---|---|
| | |

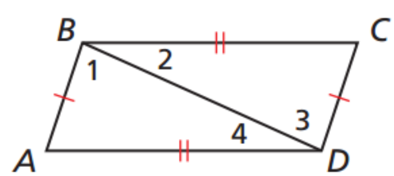
### PROVE THIS PROPERTY:

- If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

3. Given: $\overline{AB} \cong \overline{CD}$

   $\overline{BC} \cong \overline{DA}$

   Prove: *ABCD* is a parallelogram



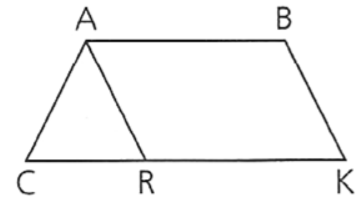

| STATEMENTS | REASONS |
|---|---|
| | |

4. Given: $\triangle CAR$ is isosceles w/base $\overline{CR}$

$\overline{AC} \cong \overline{BK}$

$\angle C \cong \angle K$

Prove: $BARK$ is a parallelogram




| STATEMENTS | REASONS |
| --- | --- |
| | |

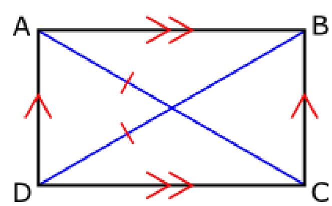
# 6.4 – Rectangles, Rhombi, & Squares

Objectives:

- Apply the properties of rectangles, rhombi, and squares to find side lengths, segment lengths, and angle measures
- Find areas of rectangles, rhombi, and squares

- Special Parallelograms
  - Rectangle
    - A quadrilateral with opposite sides congruent and with four right angles
  - Rhombus
    - A quadrilateral with all sides congruent
  - Square
    - A quadrilateral with four right angles and all sides congruent

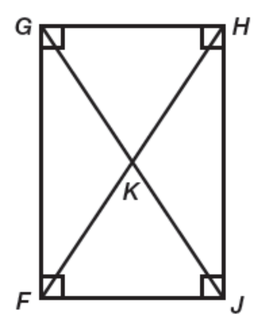
- Properties of Rectangles
  - All properties of a parallelogram apply:
    - Both pairs of opposite sides are parallel & congruent
    - Both pairs of opposite angles are congruent
    - Consecutive angles are supplementary
    - The diagonals bisect each other
  - All angles are right angles
  - The diagonals are congruent




$$A = bh$$

For rectangle *FGHJ*, the following statements are true.

- $\overline{FG} \cong \overline{HJ}$ and $\overline{FJ} \cong \overline{GH}$
- $\overline{FG} \parallel \overline{HJ}$ and $\overline{FJ} \parallel \overline{GH}$
- $\overline{FH} \cong \overline{GJ}$
- $\overline{GK} \cong \overline{JK}$ and $\overline{FK} \cong \overline{HK}$



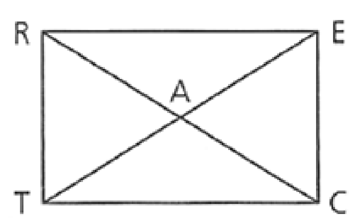

## Examples: Using the Properties of Rectangles

1. Given: *RECT* is a rectangle

   $RA = 43x$

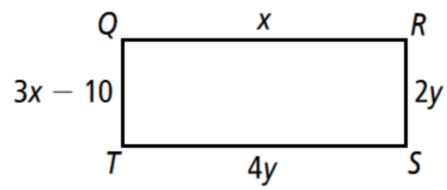
   $AC = 214x - 742$

   Find: The length of $\overline{ET}$ to the nearest tenth

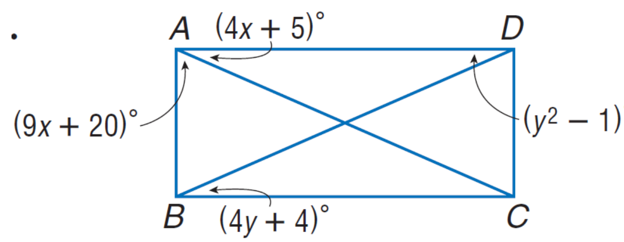


## EXAMPLES: USING THE PROPERTIES OF RECTANGLES

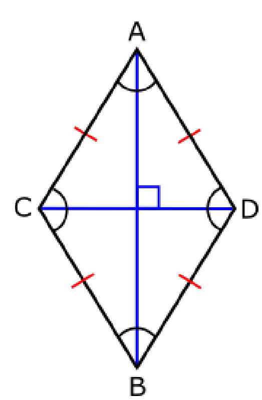
2. Given: Rectangle *QRST*
    a. Set up and solve a system of equations to find the value of the variables.
    b. Find the rectangle's base and height.
    c. What is the area of rectangle *QRST*?




3. Given: Rectangle *ABCD*
    a. Find the values of $x$ and $y$.
    b. Find $m\angle BAC$ & $m\angle DBC$.



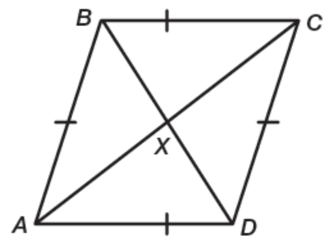

- Properties of Rhombuses
    - All properties of a parallelogram apply
        - Both pairs of opposite sides are parallel & congruent
        - Both pairs of opposite angles are congruent
        - Consecutive angles are supplementary
        - The diagonals bisect each other
    - All sides are congruent—that is, a rhombus is equilateral
    - The diagonals bisect the vertex angles
    - The diagonals are perpendicular bisectors of each other
    - The diagonals divide the rhombus into four congruent right triangles




$$A = \frac{1}{2}d_1 d_2$$

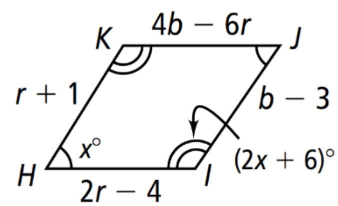
For rhombus *ABCD*, the following statements are true:

- $\angle ABC \cong \angle CDA$ and $\angle BCD \cong \angle DAB$
- $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{DA}$
- $\overline{AC} \perp \overline{BD}$
- $\overline{AX} \cong \overline{CX}$ and $\overline{BX} \cong \overline{DX}$
- $\angle BAC \cong \angle DAC$, $\angle ABD \cong \angle CBD$, $\angle BCA \cong \angle DCA$, and $\angle CDB \cong \angle ADB$

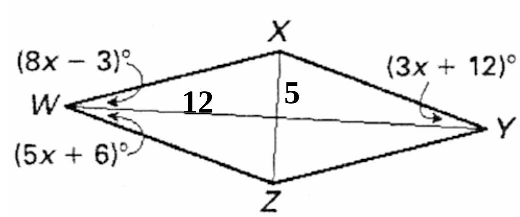



## EXAMPLES: USING THE PROPERTIES OF RHOMBUSES

4. Given: Rhombus *HIJK*
   a. Find the value of the variables.
   b. Find $m\angle J$ & $m\angle K$.
   c. What is the perimeter of rhombus *HIJK*?

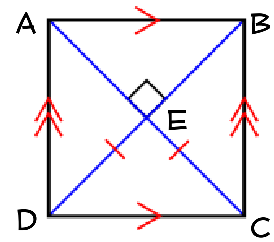



5. Given: Rhombus *WXYZ*
   a. Find the value of $x$.
   b. Find the area of rhombus *WXYZ*.
   c. Find the *WX*. What is the perimeter of *WXYZ*?



❖ Properties of Squares

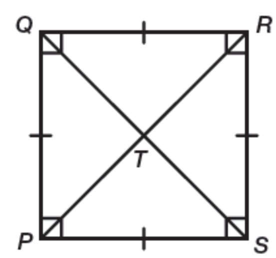
- All the properties of a parallelogram apply:
  - Both pairs of opposite sides are parallel & congruent
  - Both pairs of opposite angles are congruent
  - Consecutive angles are supplementary
  - The diagonals bisect each other
- All the properties of a rectangle apply:
  - All angles are right angles
  - The diagonals are congruent
- All the properties of a rhombus apply:
  - All sides are congruent
  - The diagonals bisect the vertex angles
  - The diagonals are perpendicular bisectors of each other
- The diagonals form four isosceles right triangles




$$A = s^2$$

For square *PQRS*, the following statements are true:

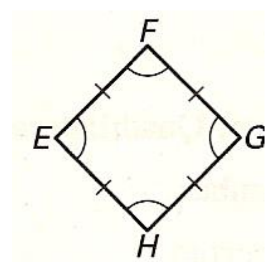
- $\overline{PR} \cong \overline{QS}$
- $\overline{PQ} \parallel \overline{RS}$ and $\overline{PS} \parallel \overline{QR}$
- $\overline{PT} \cong \overline{RT}$ and $\overline{QT} \cong \overline{ST}$
- $\angle PQS \cong \angle RQS$, $\angle QRP \cong \angle SRP$, $\angle RSQ \cong \angle PSQ$, and $\angle SPR \cong \angle QPR$
- $\overline{PR} \perp \overline{QS}$




## EXAMPLES: USING THE PROPERTIES OF SQUARES

6. Given: *EFGH* is a square with a perimeter of 36
   $EH = x + 6$
   $\angle F = 2y - 4$

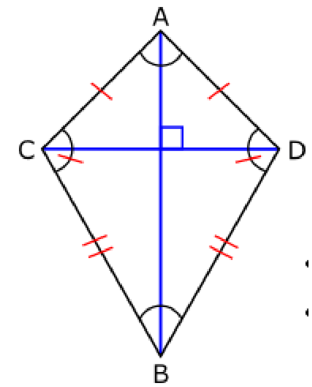
   Find: $x$ & $y$
   The area of square *EFGH*



# 6.5 – Kites & Trapezoids

Objectives:

- Apply the properties of kites and trapezoids to find side lengths, segment lengths, and angle measures
- Find areas of kites and trapezoids

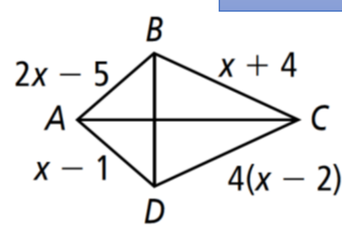



- Kites
  - A quadrilateral with two pairs of consecutive congruent sides with opposite sides that are NOT congruent.
- Properties of Kites
  - The diagonals are perpendicular to each other
  - One diagonal is the perpendicular bisector of the other
  - One of the diagonals bisects a pair of opposite angles
  - One pair of opposite angles are congruent

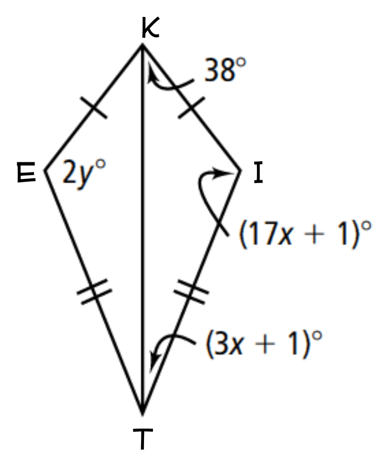
$$A = \frac{1}{2}d_1d_2$$

## EXAMPLES: USING THE PROPERTIES OF KITES

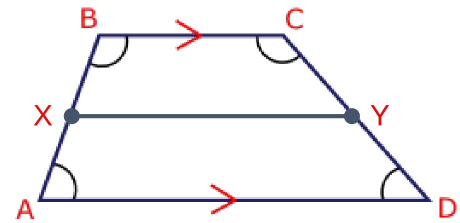
1. Given: Kite *ABCD*
   a. Find the value of *x*.
   b. Find the perimeter of *ABCD*.




2. Given: Kite *KITE*
   a. Find the values of *x* and *y* in the kite shown.
   b. $KE = 2n^2 + 29n - 33$ & $KI = 10n$. Set up and solve a quadratic equation to find the value of *n* (that makes sense).



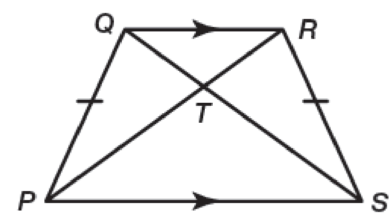
- Trapezoids
  - A quadrilateral with exactly one pair of parallel sides.
    - $\overline{BC} \parallel \overline{AD}$




- Properties of Trapezoids
  - Consecutive non-base angles are supplementary.
    - $\angle A$ is supplementary to $\angle B$
    - $\angle C$ is supplementary to $\angle D$
- Midsegment of a Trapezoid
  - Parallel to the bases
  - Length is half the sum of the lengths of the bases:

$$XY = \frac{1}{2}(AD + BC)$$

- Isosceles Trapezoids
  - A trapezoid with congruent non-parallel sides (legs)
    - $\overline{QP} \cong \overline{RS}$

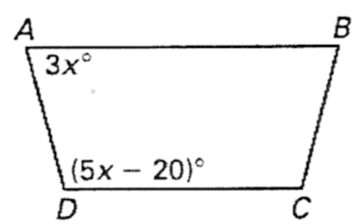



- Properties of Isosceles Trapezoids
  - All properties of a trapezoid apply
  - The base angles are congruent.
    - $\angle QPS \cong \angle RSP$
    - $\angle PQR \cong \angle SRQ$
  - The diagonals are congruent.
    - $\overline{QS} \cong \overline{RP}$

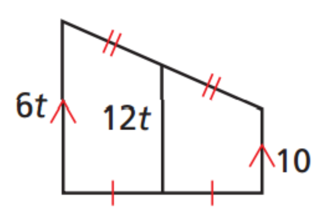
$$\boldsymbol{A = \frac{1}{2}h(b_1 + b_2)}$$

## EXAMPLES: USING THE PROPERTIES OF TRAPEZOIDS

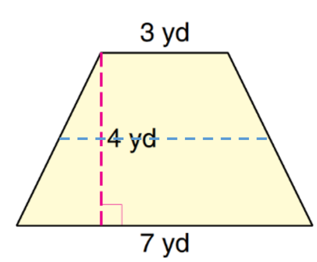
3. *ABCD* is a trapezoid. Find the value of *x*.
   Find $m\angle A$ & $m\angle D$.



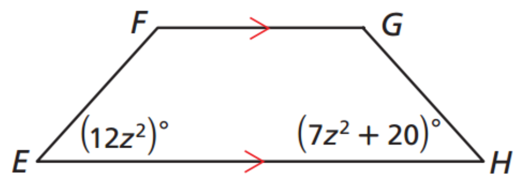

4. Find the value of *t* and the length of the midsegment of the trapezoid shown:




5. Find the length of the midsegment AND the area of the trapezoid shown:



6. Given: Isosceles trapezoid *EFGH*
   a. Find the value of $z$.
   b. If $EF = 2x^2 - 6x - 99$ & $HG = 7x$, find the value of $x$ that makes sense. Then find *EF*.



# 6.6 – Quadrilaterals in the Coordinate Plane

Objective:

- Use the distance, slope, and midpoint formulas to prove that a figure graphed in the coordinate plane is special quadrilateral: rectangle, rhombus, square, kite, or trapezoid

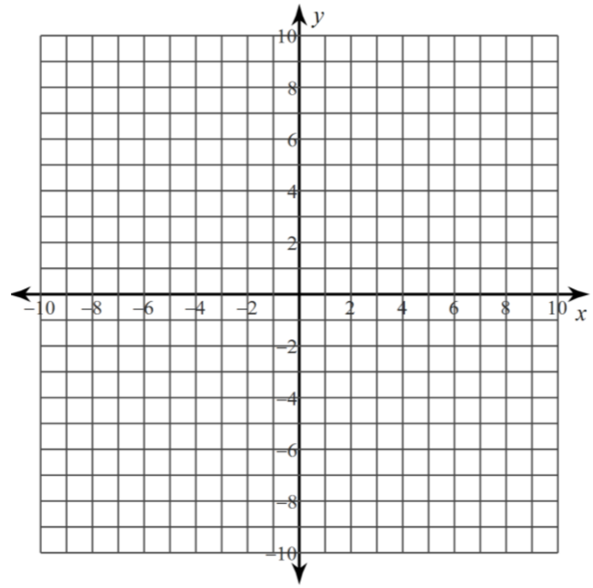
| Formulas & the Coordinate Plane | |
|---|---|
| FORMULA | WHEN TO USE IT |
| Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ | To determine whether… <br>• Sides are congruent <br>• Diagonals are congruent |
| Midpoint Formula: $(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ | To determine… <br>• The coordinates of a midpoint of a side <br>• Whether diagonals bisect each other |
| Slope Formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$ | To determine whether… <br>• Opposite sides are parallel <br>• Diagonals are perpendicular <br>• Sides are perpendicular |

| Quadrilateral | Prove: |
|---|---|
| Parallelogram | • Both pairs of opposite sides are parallel <br>• Both pairs of opposite sides are congruent <br>• One pair of opposite sides are parallel and congruent <br>• Diagonals bisect each other |
| Rectangle | First prove it's a parallelogram, and then prove… <br>• The diagonals are congruent <br>• Two consecutive sides of the parallelogram are perpendicular |
| Rhombus | First prove it's a parallelogram, and then prove… <br>• Two consecutive sides are congruent <br>• The diagonals are perpendicular <br>OR… <br>• All four sides are congruent |
| Square | • It's a rectangle <u>and</u> a rhombus (see above) |
| Trapezoid | • Only one pair of sides are parallel |

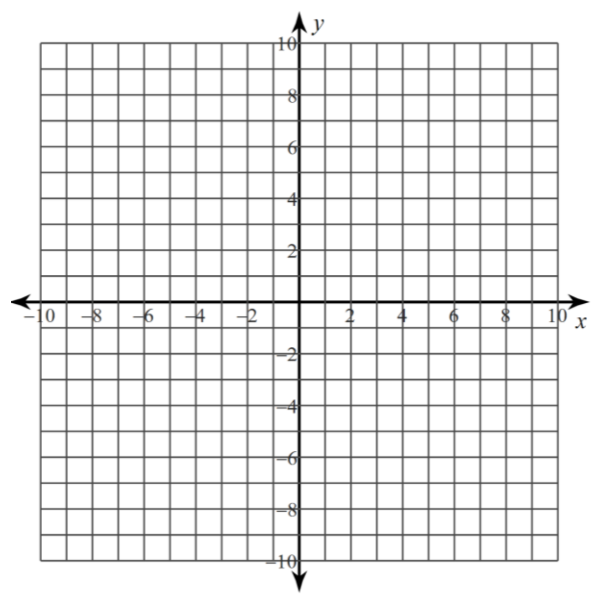
## EXAMPLES: QUADRILATERALS IN THE COORDINATE PLANE

1. The vertices of *KARI* are $K(2,1)$, $A(4,4)$, $R(10,0)$, & $I(8,-3)$. Show that *KARI* is a rectangle.

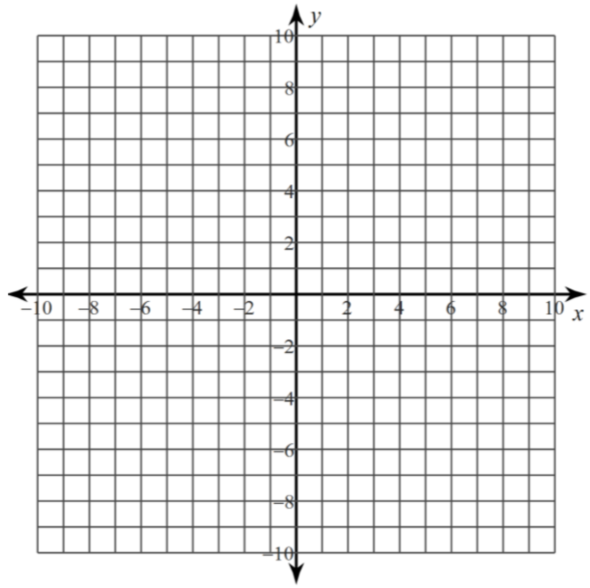
   (Remember, you must first show that *KARI* is a parallelogram.)




2. The vertices of *DION* are $D(-2,1)$, $I(1,7)$, $O(8,7)$, & $N(5,1)$. Prove that *DION* is a parallelogram. Is *DION* a rhombus?



3. Quadrilateral *JACK* has vertices $J(1, -4)$, $A(10, 2)$, $C(8, 5)$, & $K(2, 1)$. Prove that *JACK* is a trapezoid.



## DAY 2

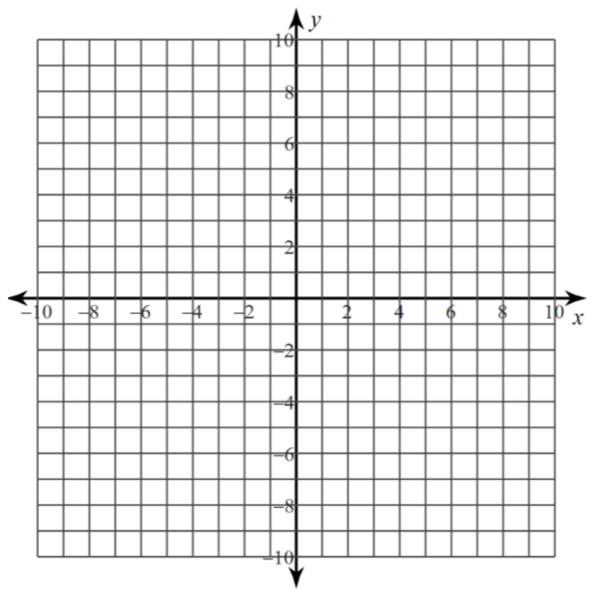
4. What special quadrilateral is formed by the intersection of these lines?

$$y = -\frac{3}{2}x + 3$$

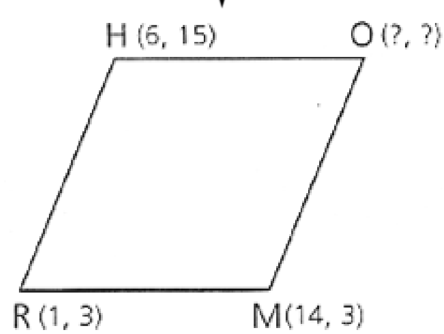
$$y = \frac{3}{2}x - 3$$

$$y = -\frac{3}{2}x + 9$$

$$y = \frac{3}{2}x + 3$$

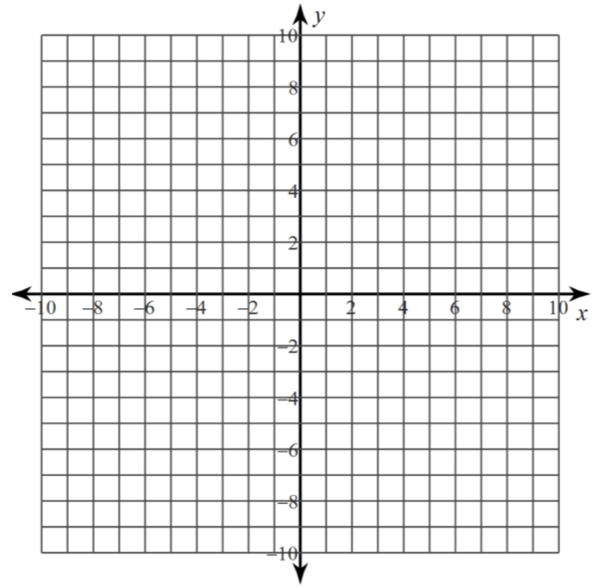


5. The coordinates of three vertices of parallelogram *RHOM* are given. Find the coordinates of *O* so that a rhombus is formed.

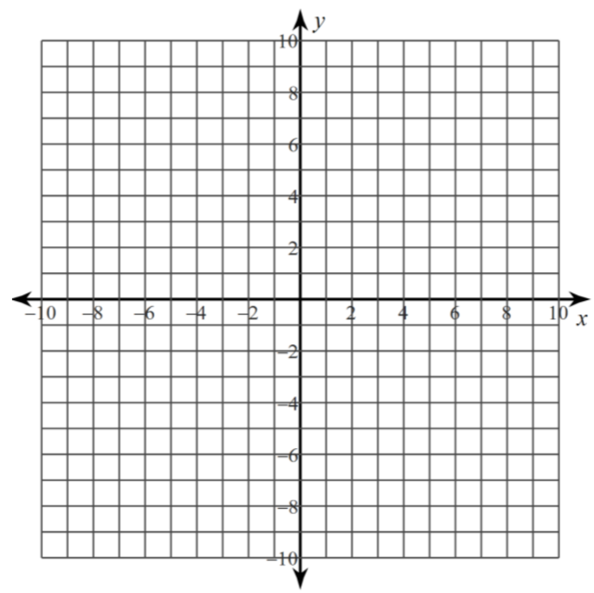


Use the <u>diagonals</u> to determine whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.

6. $A(0,2), B(3,6), C(8,6), D(5,2)$




7. $E(-4,-1), F(-3,2), G(3,0), H(2,-3)$



# 6.7 – COORDINATE PROOFS

BONUS TOPIC

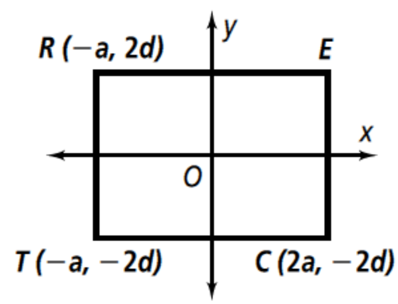
Objective:

- Use the distance, slope, and midpoint formulas in a coordinate proof to show that a figure graphed in the coordinate plane is special quadrilateral

*You can use coordinate geometry and algebra to prove theorems in geometry. This kind of proof is called a coordinate proof. Sometimes it's easier to show that a theorem is true by using a coordinate proof rather than a standard deductive proof.*

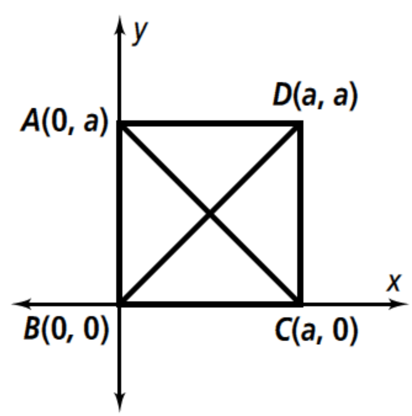
| | Distance Formula | Midpoint Formula | Slope Formula |
|---|---|---|---|
| **Formula** | $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ | $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ | $m = \frac{y_2 - y_1}{x_2 - x_1}$ |
| **When to Use It** | To determine whether<br>• sides are congruent<br>• diagonals are congruent | To determine<br>• the coordinates of the midpoint of a side<br>• whether diagonals bisect each other | To determine whether<br>• opposite sides are parallel<br>• diagonals are perpendicular<br>• sides are perpendicular |

1. Rectangle *RECT* is shown at the right. What are the coordinates of point *E*?

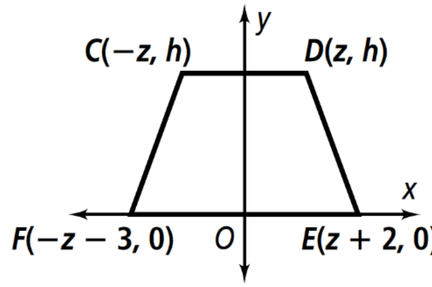



2. Use coordinate geometry to prove that ∠*T* is a right angle.

3. Given: Square *ABCD*
Prove that the diagonals are congruent and perpendicular using coordinate geometry.



4. Write a coordinate proof to prove that quadrilateral *CDEF* is an isosceles trapezoid.




5. Write a coordinate proof to prove that quadrilateral *ABCO* is a parallelogram.